Uncertainties in the Prediction of Design Waves on Shallow Foreshores of Coastal Structures

Markus Muttray[1], Hocine Oumeraci[2] und Matthias Bleck[3]

Abstract: Based on extensive hydraulic model testing, the uncertainties of available „standard“ models used to predict wave heights on shallow foreshores of coastal structures are quantified. Both wave transformation on the foreshore and at the structure are taken into account. Finally, improvements of the prediction models are proposed and recommendations on the selection of the models to be used are provided.

INTRODUCTION

One of the prerequisites to help moving sustainable design of coastal protection from an academic debate into the realm of concrete work, performance and return is the development and use of probabilistic design tools (OUMERACI; 2000). However, the implementation of a reliability based design implies among others that the uncertainties associated with the prediction of wave conditions at the design site should be reliably assessed. In fact, small errors in estimating design waves may result in much larger errors for thep redicted wave loads, wave overtopping, structure stability, etc.. One of the main results of a large European research project on reliability based design of coastal structures was that most uncertainties still originate from the errors in predicting wave transformation from deep water towards shallow foreshores (OUMERACI et al.; 2001). Therefore, a basic research project was initiated by the authors to determine the uncertainties associated with the prediction of wave transformation on shallow foreshores and in presence of coastal structures with different reflection properties. The project has just been completed and the results of

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[1] Senior Coastal Engineer; Delta Marine Consultants bv; P.O. box 63; 2800 AB Gouda, The Netherlands; mmuttray@dmc.nl

[2] Professor and Head of Leichtweiß-Institute for Hydraulic Engineering; Beethovenstr. 51a; 38106 Braunschweig, Germany; h.oumeraci@tu-bs.de

[3] Teaching and Research Assistant; Leichtweiß-Institute; m.bleck@tu-bs.de

the whole project have been summarized in a final report (OUMERACI and MUTTRAY, 2001).

This paper is intended to principally discuss some of the results by focusing on the uncertainties associated with the prediction of wave heights on shallow foreshores and in the presence of coastal structures.

EXPERIMENTAL SET-UP AND PROCEDURE

Hydraulic model tests using both regular and irregular waves have been conducted in the wave flume (100 m x 2 m x 1.25 m) of the Leichtweiss-Institute (LWI) in Braunschweig, Germany. A detail of the tested foreshore geometry is shown in Fig. 1. A total of 14 waves gauges were installed to record the waves in the far field and in the foreshore. A run-up gauge was also installed to record the water surface elevation directly at the sloping structures (not shown in Fig. 1). For the irregular wave tests trains of 100 waves were used for analysis. The water depth at the upper end of the foreshore was varied from h = 10 cm to h = 30 cm, the wave height from H_s = 5 cm to H_s = 25 cm and the wave period from T_p = 1.25 s to T_p = 3.55 s.

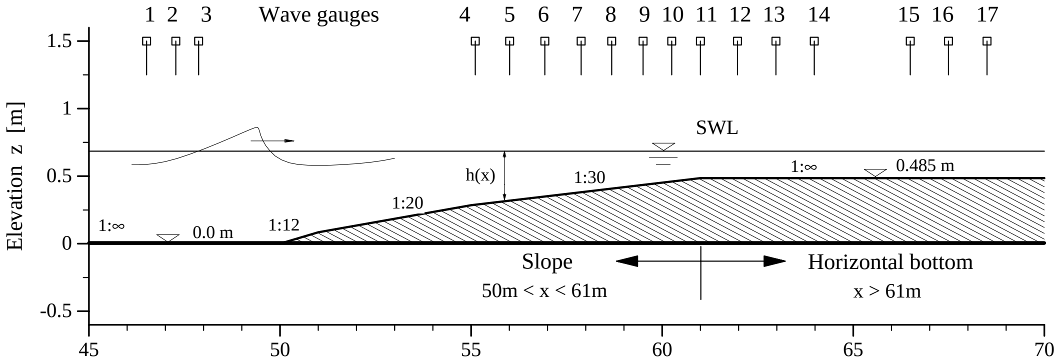

Fig. 1. Experimental Set-up and location of the wave gauges (without sloping structures)

The main objective of the hydraulic model tests is to quantify the uncertainties associated with the prediction of wave shoaling, wave height at breaking inception and after breaking, and then to draw tentative recommendations on the prediction models to be used. In addition, the effect of wave set-up and wave reflection on wave shoaling and wave breaking is also addressed.

Experimental Results and Discussion

Prediction Models, Calculations Procedure and Definition of the Uncertainties

The prediction models used to calculate the wave transformation on shallow foreshores (engineering approach) are described by MUTTRAY and OUMERACI (2000) and in more detail by OUMERACI and MUTTRAY (2001). A brief summary of these models is given in Plate 1. Regarding the wave damping due to friction at the side walls and at the bottom of the flume it should be stressed that this does not affect the local wave height in the surf zone, because the wave energy dissipation due to wave breaking already covers these losses.

1. Wave Set-Up

$$\frac{d\bar{\eta}}{dx} = \frac{1}{\bar{\eta}+h}\cdot\frac{d}{dx}\left[\frac{1}{8}\overline{H^2}\left(\frac{1}{2}+\frac{2kh}{\sinh 2kh}\right)\right] \qquad (1)$$

$\bar{\eta}$ = wave set-up; $\bar{H}$ = mean wave height; h = water depth; k = (2 π) / L = wave number

2. Wave Shoaling

Non-linear shoaling koefficient k_s according to explicit approximation by OUMERACI & MUTTRAY (2001) of exact solution by SHUTTO (1974) using cnoidal theory:

$$\frac{K_s}{K_{sA}} = 1 \ \text{(linear)} \qquad \text{for } \Pi < \frac{2}{15}$$

$$\frac{K_s}{K_{sA}} = 1 + 0.4\left[\Pi - \frac{2}{15}\right]^{5/4} \qquad \text{for } \frac{2}{15} < \Pi < 0.5 \qquad (2)$$

$$\frac{K_s}{K_{sA}} = 0.69 + 0.6\ \Pi^{1/2} \qquad \text{for } \Pi > 0.5$$

with non-linearity parameter:

$$\Pi = \frac{H}{L}\coth^3 kh \qquad (3)$$

and linear shoaling coefficient:

$$K_{sA} = \left[\left(1 + \frac{2kh}{\sinh\ 2kh}\right)\ \tanh\ kh\right]^{-1/2} \qquad (4)$$

Remark: Eq.(2) deviates by less than 2% from exact solution by SHUTTO (1974)

3. Wave Breaking

3.1 Theoretical Breaking Criterion of MICHE (1944) derived for regular waves (horizontal bottom):

$$\frac{H_b}{L} = M\ \frac{L}{L_0} = M\ \tanh\ kh \qquad (5)$$

with: H_b = Breaker Height
L, L_0 = local and deep water wave length

3.2 Empirical breaking criterion of WEGGEL (1972) for regular waves over sloping bottom:

$$\frac{H_b}{h_b} = b - a\frac{H}{gT^2} \qquad (6)$$

with:

$$a = 43.8(1 - \exp(-19\tan\ \alpha))$$

$$b = \frac{1.56}{1+\exp(-19.5\tan\alpha)}$$

3.3 Semi-empirical breaking-criterion of GODA (2000) for regular and irregular waves over sloping bottom:

$$\frac{H_b}{L_0} = A\left[1 - \exp\left(1 - \exp\ (-1.5\frac{\pi h}{L_0}\ (1 + 15\ \tan^{3/4}\alpha))\right)\right] \qquad (7)$$

with: A = 0.17 for regular waves and A = 0.12 to 0.18 for irregular waves

4. Wave Damping due to Friction:

Formula by IWAGAKI and TSUCHIYA (1966) using linear wave theory and a laminar boundary layer at the bottom and on the side walls of the wave flume:

$$K_d = \frac{H(x)}{H_0} = \exp\left[-\sqrt{\frac{\nu T}{\pi}}\left(1 + \frac{1}{\kappa_{bw}}\right)\frac{k^2 x}{2kh + \sinh\ 2kh}\right] \qquad (8)$$

Plate 1. Prediction Models for Wave Transformation on Shallow Foreshores (Engineering Approach)

As already reported by MUTTRAY and OUMERACI (2000) two types of calculation procedures have been used to predict the local wave height in the surf zone (see Plate 1).

- *Type 1:* a simplified procedure
  taking only wave shoaling (linear or non-linear, Eq.(4) od Eq.(2)) and wave breaking (Eq.(5), Eq.(6) or Eq.(7)) into account ($H_{(x)} = f(K_s, H_{crit})$);
- *Type 2:* a more complete procedure
  taking wave shoaling, wave damping (Eq.(8)), wave set-up (Eq.(1)) and wave breaking into account ($H_{(x)} = f(K_s, K_d, \bar{\eta}, H_{crit})$).

The uncertainties of the predicted local wave parameters are described by the standard deviation, the coefficient of variation (Co V) and the systematic deviation between calculated values x_i (prediction) and expected values E_x (measurements) which are defined in MUTTRAY and OUMERACI (2000).

Wave Shoaling, Wave Breaking and Post-Breaking Wave Heights

The main findings on the uncertainties associated with the prediction of local wave heights which have recently been reported by MUTTRAY an OUMERACI (2000) are first discussed before embarking into a more detailed discussion on the effect of wave set-up and wave reflection.

Uncertainties in Wave Shoaling Prediction (Before Breaking)

The main results are summarized in Table 1 in terms of the coefficient of variation σ' of the shoaling coefficient associated with the significant (H_s) and the maximum (H_{max}) wave height of an irregular wave train as well as with the height H_m of regular waves. Focus is put on the results obtained by using the simplified procedure. The figures in parentheses are related to the results by using the complete calculation procedure.

Table 1. Uncertainties in shoaling coefficient predicted by linear and non-linear shoaling models (see Plate 1)

Type of Waves	Wave Height	Coefficient of Variation σ' for:	
		Linear Model (Eq. 4)	Non-linear Model (Eq. 2)
Irregular Waves	H_s	σ'_{Hs} = 3.9 % (3.8)*	σ'_{Hs} = 10.4 % (9.4)*
	H_{max}	σ'_{Hmax} = 12.7 % (13.5)*	σ'_{Hmax} = 6.1 % (6.5)*
Regular Waves	H_m	σ'_{Hm} = 14.7 % (15.6)*	σ'_{Hm} = 8.7 % (9.3)*

*Figures in ( ) are related to complete calculation procedure (type 1).

First, it is important to stress that the uncertainties are not or not significantly reduced by considering the additional effects which are accounted for by the complete calculation procedure. Therefore, the discussion is focused only on the results obtained by using the simplified calculation procedure.

Local significant wave heights H_s are better predicted by the linear shoaling model (Eq. 4) with an uncertainty of $\sigma'_{Hs} \approx 4$ %. Using a non-linear model (Eq. 2) would dramatically increase the uncertainty ($\sigma'_{Hs} \approx 10$ %).

Considering the maximum wave height H_{max} in an irregular wave train a better prediction is provided by the non-linear model (Eq. 2) with an uncertainty of $\sigma'_{Hmax} \approx 6$ %. Using the linear model would result in a much higher uncertainty ($\sigma'_{Hmax} \approx 13$ %).

A possible implication of the different shoaling behaviour of H_s and H_{max} is that the statistical distribution of wave heights in a spectrum will be affected when the waves enter shallow water, i.e. even before any wave breaking starts.

Surprisingly, the uncertainties associated with the predicted height H_m of regular waves are much less when using the non-linear shoaling model ($\sigma'_{Hm} \approx 9$ %) than the linear model ($\sigma'_{Hm} \approx 15$ %).

Uncertainties in the Predicted Wave Height at Breaking

Considering wave breaking on shallow foreshores, it was found that the local wave height H_b at the breaking point is essentially governed by the shoaling process. Accounting for further processes as this is done in the complete calculation procedure (type 2) generally leads to an uncertainty reduction of less than $\Delta\sigma' = 1$ %. Overall, the GODA-model (Eq. 7) proved to be more appropriate for the prediction of maximum wave height H_{max} at the breaking point by using a coefficient A = 0.15. The associated uncertainty is $\sigma' \approx 7$ %, while it is $\sigma' = 32$ % for the WEGGEL-model and $\sigma = 11$ % (with M = 0.14) for the MICHE-model.

Surprisingly, for the prediction of the significant wave height at the breaking point the GODA-model appears to be less appropriate (A = 0.10 $\sigma' \approx 17.5$ %) than the simple breaking criterion $\gamma_b = H_s/h_b = 0.5$ ($\sigma' \approx 12$ %) which does not account for the foreshore slope.

For the prediction of the breaking of regular waves the GODA-model with A = 0.17 is associated with much less uncertainty ($\sigma' \approx 9$ %) than the WEGGEL-model ($\sigma' \approx 16\%$) (MUTTRAY and OUMERACI, 2000).

Uncertainties in the Predicted Post-Breaking Wave Height

For the regular waves the local wave height $H_{(x)}$ after breaking is expressed as a function of the critical wave height H_{crit} which is predicted by the GODA-model (Eq.7):

$$H_{(x)} = H_{(x)} = H_{crit} \left[0.4 \, \exp \left(-0.15 \, \frac{\Delta_x}{h_{(x)}} \right) + 0.6 \right] \qquad (9)$$

where x = distance from the breaking point; h(x) = water depth at distance x and 0.6 $H_{crit} = H_{stable}$ is the stable wave height which prevails after a certain distance from the breaking point. Eq. (9) which is associated with uncertainties in the order of $\sigma' = 15$ % exhibits a similar experimental decay as the empirical relationship obtained by KWEON and GODA (1996) by re-analysing available data from model tests with regular waves. In contrast to the findings of KWEON and GODA (1996) no distinct influence of the wave period could be found (MUTTRAY and OUMERACI, 2000). Moreover, much simpler models than that suggested by KWEON and GODA (1996) have been derived for random waves (MUTTRAY and OUMERACI, 2000):

- Maximum wave height: $H_{max}(x) = H_{crit}$ (10)
  with σ' = 9 % and H_{crit} according to Eq. (7)

- Significant wave height: $H_s(x) = 0.5\ h(x)$ (11)
  with σ = 13 % and $h_{(x)}$ = local water depth.

Effect of Wave Set-Up

The comparison between the directly measured wave set-up $\overline{\eta}(x)$ and the wave set-up $\overline{\eta}$ ($H_m(x)$) calculated by using Eq. (1) and the locally measured wave height ($H_m(x)$ in Fig.2) shows that the spatial distribution of the wave set-up is qualitatively well-described by the calculation while quantitatively some differences occur which are particularly significant immediately behind the breaking point.

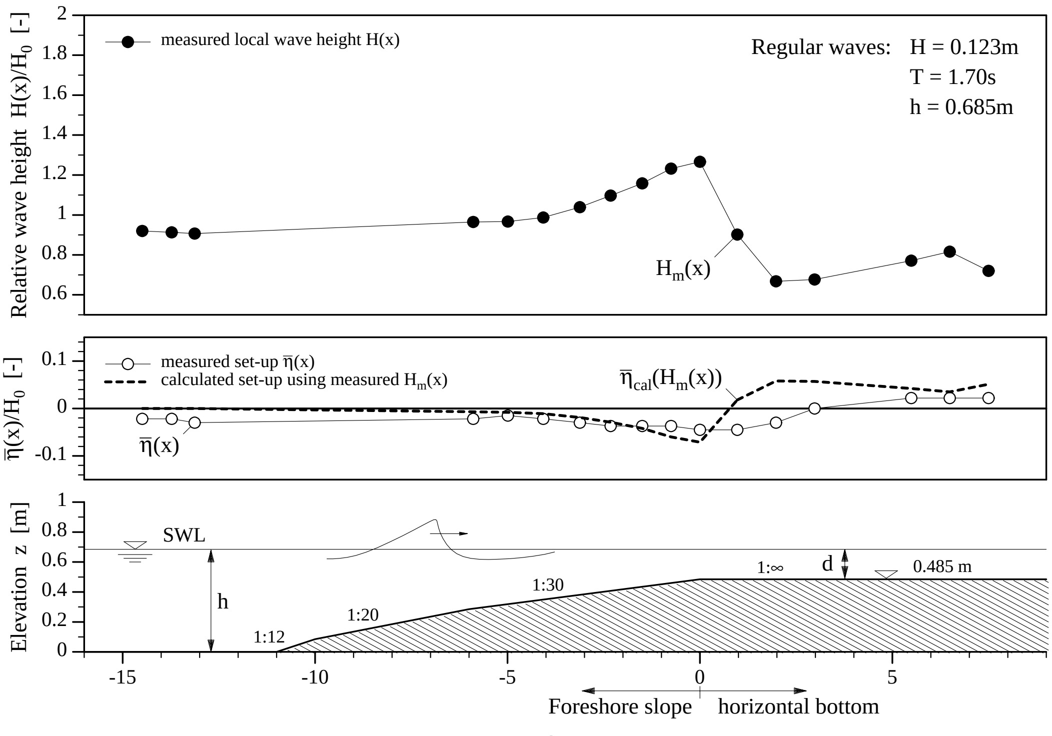

Fig. 2 Cross-shore distribution of wave height and wave set-up

By considering all the data obtained from regular wave tests and relating the wave set-up $\overline{\eta}_{meas}(x)$ by the local water depth h(x) at the same location x across shore a more detailed comparison with the calculated set-up values $\overline{\eta}_{cal}(x)$ is drawn in Fig.3, showing that the calculaion (Eq.1) by using measured local wave heights H(x) slightly underestimates the set-down before breaking while the set-up after breaking is overestimated.

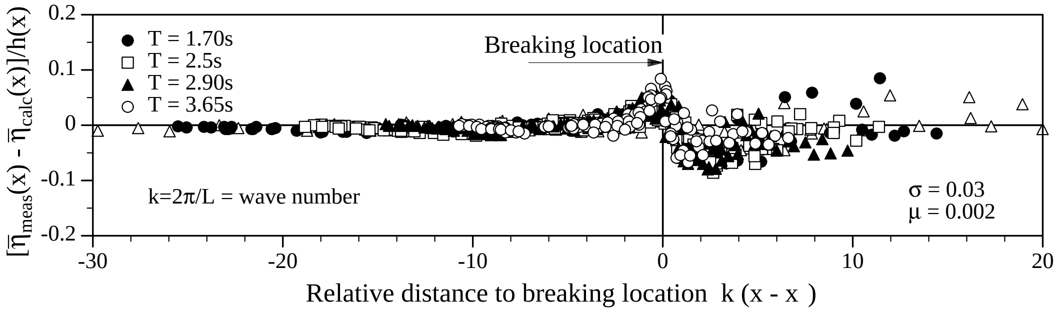

Fig.3 Comparison between measured and calculated relative wave set-up

A direct comparison of the calculated and measured wave set-up for both regular and irregular waves has led to a coefficient of variation of σ’ ≈ 50 % for regular waves and σ’ ≈ 60 % for irregular waves.

Overall, the tests with irregular waves resulted in much smaller set-up values $\bar{\eta}(x)$ than those with regular waves. The reasons for this difference may be seen from Fig.4, showing the set-up η(t) recorded at wave gauge 15 (see Fig. 1) located at x = 5.49 m after the breaking point for a regular and an irregular wave test with approximately similar conditions. While the set-up for regular waves remains at a high and relatively constant level over the test duration, it is very variable for irregular waves. Instantaneous values η(t) can even be higher than for regular waves.

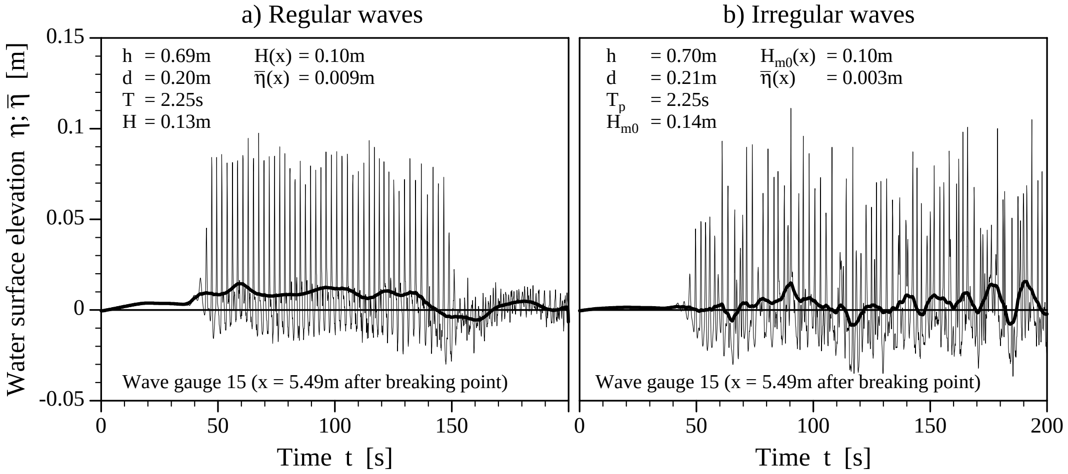

Fig.4 Recorded wave set-up for regular and irregular waves

As previously mentioned the wave attenuation due to friction does not affect the local wave height after breaking. If we assume that friction has no significant effect up to the point of breaking, then the effect of wave set-up on the local wave height along the entire cross-shore profile can be found through comparison of the results obtained by calculation procedure type 1 (simplified) and calculation procedure type 2 (complete). Based on such results and for the range of the set-up values measured in this study the following tentative conclusions can be drawn on the effect of the wave set-up on the prediction of local wave heights:

- before and at breaking: no significant effect could be identified,
- after breaking: no significant effect could be identified for the predicted regular wave heights and for significant wave heights, but considering wave set-up reduces the uncertainty in the prediction of maximum heights from σ' = 14 % to σ' = 9 %.

Effect of wave reflection

The effect of wave reflection on wave shoaling and wave breaking, and thus on the local wave height in front of structures, was examined by comparative analysis of the same model tests performed with the foreshore alone (see Fig. 1) and with different reflecting structures added at various locations along the horizontal bottom behind the foreshore slope. First results are discussed below examplarily for an impermeable vertical non-overtopped wall located at the upper end of the foreshore slope.

The comparison in Fig.5 of the incident wave height obtained through (i) reflection analysis at the wall ($H_{i,wall\ (with\ wall)} = H(x)/(1+Kr)$) and (ii) direct measurement at the same location without wall ($H_{i,wall\ (without\ wall)}$) shows that wave reflection does not affect the local incident wave height and that the shoaling models (Eqs. (2) & (4)) can be used for progressive waves as well as for partially and totally standing waves.

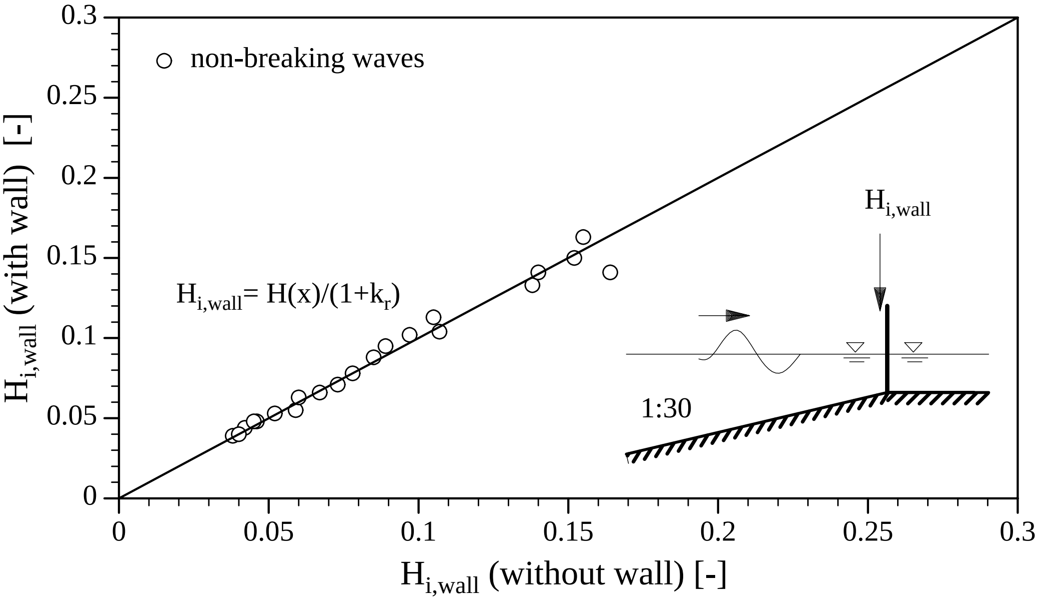

Fig.5 Incident wave height at wall location for models with and without wall

A further comparison in Fig.6 of the location of the breaking point - Δx_b (relative to wall location) with and without wall surprisingly shows that wave reflection does not significantly affect the location of the breaking point. This is contrary to the results of RUGGIERO and McDOUGAL (2001) suggesting that the reflected waves should cause the incident waves to begin breaking further offshore. Analysis of further test series will be undertaken in order to confirm this finding. As expected, however, wave reflection is found to substantially affect the breaker types.

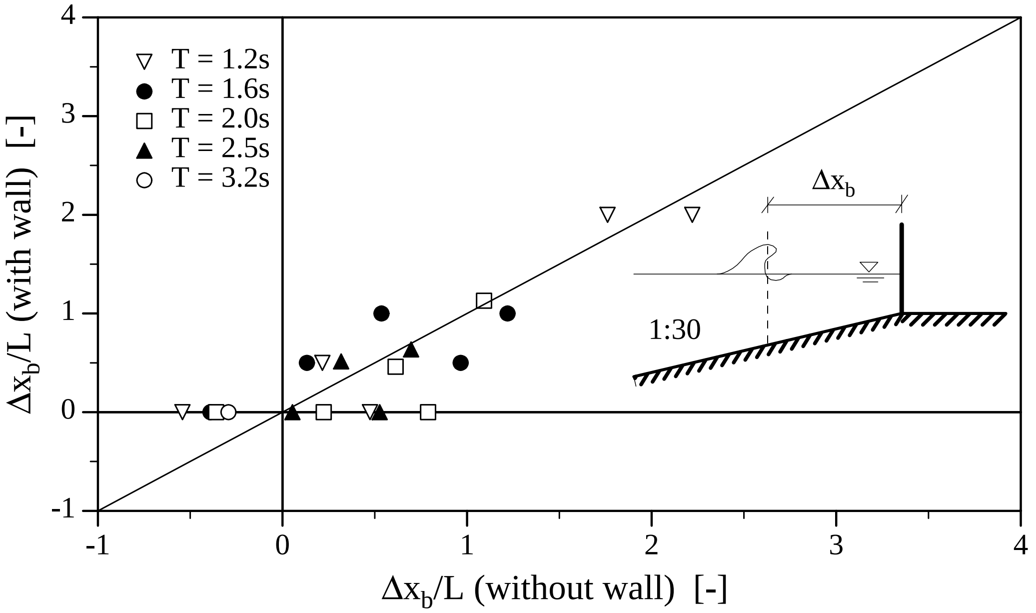

Fig.6 Effect of wave reflection on the location of the breaking point

CONCLUDING REMARKS

Prior to breaking, regular wave heights and maximum wave heights in an irregular wave train are better predicted by a non-linear shoaling model (Eq.2) resulting in uncertainties of $\sigma'_{Hm}\approx9\%$ and $\sigma'_{Hmax}\approx6$ %, respectively. For the prediction of significant wave heights a linear shoaling model (Eq.4) is more appropriate ($\sigma'_{Hs}\approx4$ %).

At breaking, the GODA-model (Eq. 7) was found to be appropriate for the prediction of regular wave heights with A = 0.17 (σ' = 9 %) and the maximum wave height in an irregular wave train with A = 0.15 (σ' ≈ 7 %). For the prediction of significant wave heights, the use of a simpler breaking criterion leads to less uncertainty than the GODA-model.

After breaking, the maximum wave height H_{max} and the significant wave height H_s can be predicted by simple relationships (Eqs. 10 & 11) resulting in a coefficient of variation of $\sigma'_{Hmax} = 9$ % and $\sigma'_{Hs} = 13$ %, respectively. For regular waves a more complicated relationship (Eq. 9) with more uncertainty ($\sigma'_{Hm} \approx 15$ %) has been derived.

Unexpectedly, no significant effect of the wave set-up could be identified for the prediction of local wave heights before, at and after breaking. A slight reduction of the uncertainty is achieved by considering wave set-up only for the prediction of maximum wave height at the breaking.

Even more unexpected are the results related to the effect of wave reflection on the local wave height. Contrary to the results reported in the literature, the reflected waves do not cause the incident waves to break further offshore, although wave reflection was observed to affect the breaker type. Moreover, the shoaling models and the breaking criteria for progressive waves remain valid in the presence of wave reflection.

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REFERENCES

Goda; Y. 1983. A unified nonlinearity parameter of water waves. *Report of the Port and Harbour Research Institute*, 22(3), 3-30.

Goda, Y. 2000. Random Seas and Design of Maritime Structures. *Advanced Series on Ocean Engineering;* Vol. 15; World Scientific.

Iwagaki, Y. and Tsuchiya, Y. 1966. Laminar damping of oscillatory waves due to bottom friction. *Proceedings International Conference Coastal Engineering* (ICCE); Vol. 10; pp. 149-174; Tokyo.

Kweon, H.-M. and Goda, Y. 1996. A parametric model for random wave deformation by breaking on arbitrary beach profiles. *Proc. 25th Intern. Conf. Coastal Eng.*, pp. 261-274; Orlando/Florida..

Miche, M. 1944. Mouvements ondulatoires de la mer en profondeur constante ou décroissante. Annales des Ponts et Chaussées; Vol. 114; pp. 25, 131, 270, 369 ff..

Muttray, M. and Oumeraci, H. 2000. Wave Transformation on the Foreshore of Coastal Structures. ASCE, Proc. ICCE 2000.

Oumeraci, H. 2000. The Sustainability Challenge in Coastal Engineering. Proc. 4th Int. Conf. of Hydrodynamics.

Oumeraci, H., Kortenhaus, A., Allsop, N.W.H., DeGroot, M.B., Crouch, R.S., Vrijling, J.K., Koortman, H.G. 2001. Probabilistic Design Tools for Vertical Breakwaters. Balkema, pp. 373.

Oumeraci, H. and Muttray, M. 2001. Bemessungswellenparameter vor Strukturen mit verschiedenen Reflexionseigenschaften. *Leichtweiss-Institute, TU Braunschweig, unpublished, Final Report,* OU 1/3-3.

Shuto, N. 1974. Nonlinear long waves in a channel of variable section. Coastal Engineering in Japan, Vol. 17, pp. 1-12, Tokyo, Japan.

Ruggiero, P. and McDougal, W.G. 2001. An analytic model for the prediction of wave wave set-up, long shore currents and sediment transport on beaches with sea walls. *Coastal Engineering*, 43 (3-4), 161-162, Elsevier.

Weggel, J.R. 1972. Maximum breaker height. *Journal of Waterways, Habours and Coastal Engineering Division*, 98(WW4), 529-548.